High Frequency Deutsche Mark-US Dollar Returns: FIGARCH Representations and Non Linearities*

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This article considers the use of the long memory volatility process, FIGARCH, in representing Deutschemark - US dollar spot exchange rate returns for both high and low frequency returns data. The FIGARCH model is found to be the preferred specification for both high frequency and daily returns data, with similar values of the long memory volatility parameter across frequencies, which is indicative of returns being generated by a self similar process. The BDS test for non-linearity is applied to the residuals of the model for the high frequency returns. No evidence is found to suggest that the procedure for filtering the high frequency returns to remove the intraday periodicity has induced any non-linearities in the residuals; and the FIGARCH specification is found to be adequate (JEL C22, F31).

Keywords: BDS test, correlation dimension, FIGARCH, high frequency data, intra day periodicity, volatility.

I. Introduction

This paper is concerned with some of the intriguing features of high frequency foreign exchange rates. In particular, we explore some

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aspects of the property of long memory, persistent volatility that has become a well documented feature of these markets; e.g. see Andersen and Bollerslev (1997b, 1998) and Dacorogna et al. (1993). We focus on the long memory volatility parameter obtained by estimating the FIGARCH model of Baillie et al. (1996) from both high and low frequency returns data. While such models have been found to provide good descriptions of daily return volatility, little is known of their adequacy in dealing with higher frequency data. Hence this paper investigates the general appropriateness of the FIGARCH specification for many different frequencies of DM-$ returns. Second, we also wish to see if the FIGARCH model is consistent with the theory that returns are a self similar process, which implies the long memory parameter is invariant to the sampling frequency; see Beran (1994). Apart from the standard diagnostic tests, the appropriateness of the FIGARCH model for 30 minute data is also investigated for the presence of further non linearities. This seems especially important given the possibility that the filtering method for removing intra day periodicity may have potentially induced spurious relationships. The estimation of the correlation dimension and the results from the BDS test fail to find any evidence of significant non-linearity in the residuals. The overall conclusion is that the FIGARCH model appears to be a good specification for the filtered returns series. An interesting implication of the robustness of the FIGARCH model and importance of the long memory volatility parameter on relatively short spans of high frequency data, strongly suggests that the long memory property is an intrinsic feature of the system rather than being due to exogenous shocks which lead to regime shifts.

The plan of the rest of this paper is as follows; section 2 discusses the application of the long memory volatility, FIGARCH model to daily and lower frequency data. This model is found to be econometrically superior to regular stable GARCH models. The FIGARCH model for the daily data is also important in constructing the filtering procedure on the 30 minute data. Section 3 discusses the basic properties of the high frequency data and the presence of long memory and intra day periodicity in the autocorrelation functions of the squared and absolute returns. The application of the Flexible Fourier Form (FFF) filter to remove deterministic intra-day periodicity is then discussed. The estimates of MA(1)-FIGARCH(1,δ,0) models for 30 minute returns, one hour and up to eight hour returns are presented. The estimates of the long memory volatility parameters are consistent with returns being generated by a self similar process. Section 4 of the paper then describes tests for non linearity, which tend to confirm the appropriateness of the filtering procedure and the use of the FIGARCH approach. Section 5 provides a brief conclusion.

II. Analysis of Low Frequency Daily Returns

This section is concerned with the analysis of daily returns from 1979 through 1998 and the estimation of a FIGARCH model to describe daily volatility. The model for daily returns provides an interesting comparison with the models for high frequency data and throws some light on the possible self similarity of DM-$ returns. Also, the model for the daily volatility process is required to filter the raw 30 minute returns to remove the strong intra day periodicity. The set of daily DM-$ spot returns used in this study were provided by the Federal Reserve Bank of Cleveland for the sample period of March 14, 1979 through December 31, 1998, which correspond to the origin of the EMS (European Monetary System), and the relaxation of capital controls. Excluding weekends and holidays, this realizes a sample of 4,989 daily observations. The autocorrelation function of the daily returns, squared returns and absolute returns are plotted in figure 1. Analogously to the 30 minute data, the autocorrelations of the squared returns and absolute returns exhibit the familiar slow, persistent decay; albeit without the strong intra day periodicity. The model that is postulated to describe the returns process is then,

$$R_t = 100\Delta \ln\left(S_t\right) = \varepsilon_t + \theta\varepsilon_{t-1}, \tag{1}$$

$$\varepsilon_t = z_t\sigma_t, \tag{2}$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \left[1 - \beta L - \left(1 - \varphi L\right)\left(1 - L\right)^{\delta}\right]\varepsilon_t^2 , \tag{3}$$

where S_t is the daily DM-$ spot exchange rate, z_t is i.i.d.(0,1) and

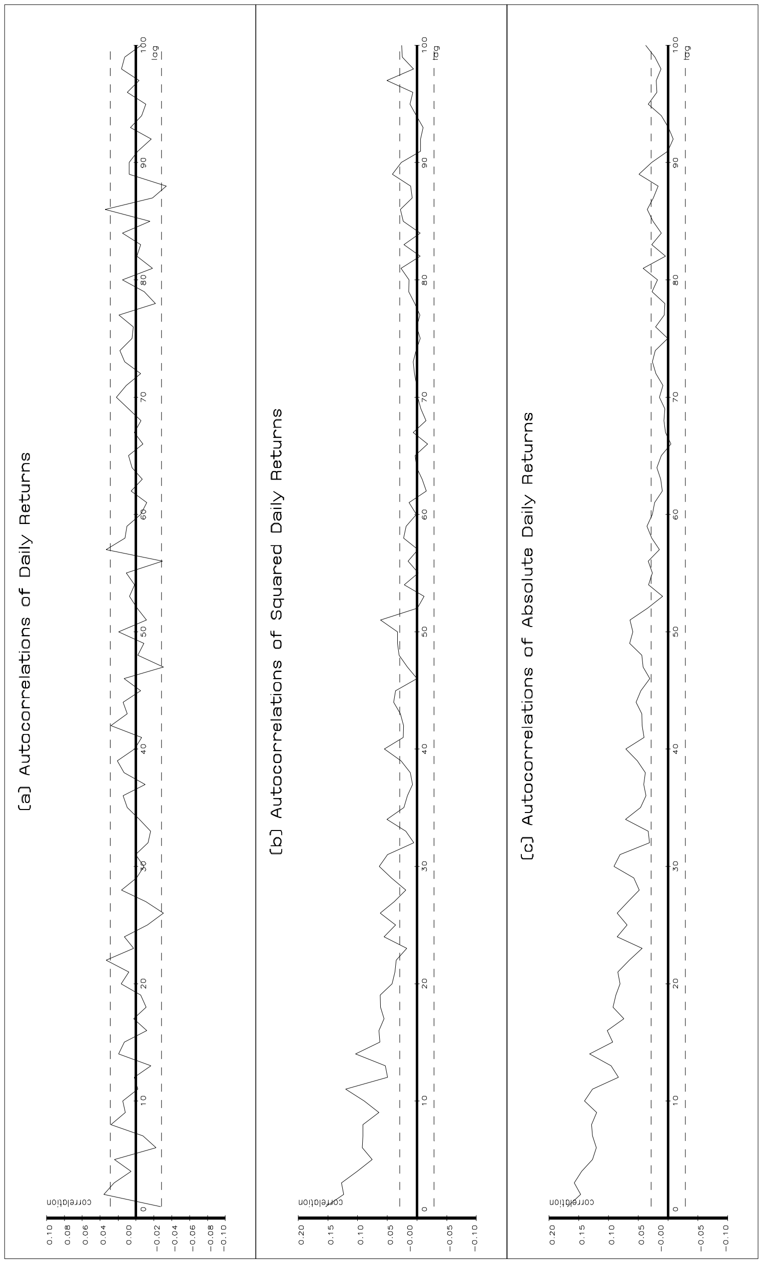

Figure 1. Correlograms of Daily Returns

returns are specified to follow an MA(1) process, while the conditional variance process σ^2_t, in equation 3, is represented by a FIGARCH (Fractionally Integrated Generalized AutoRegressive Conditional Heteroskedastic) process, as developed by Baillie et al. (1996). The above FIGARCH(1,δ,1) process is sufficiently general that it can generate very slow hyperbolic rate of decay in the autocorrelations of squared returns. When $\delta = 0$, $p = q = 1$, then equation 3 reduces to the standard GARCH(1,1) model; and when $\delta = p = q = 1$, then equation 3 becomes the Integrated GARCH, or IGARCH(1,1) model, and implies complete persistence of the conditional variance to a shock in squared returns. The FIGARCH process has impulse response weights, $\sigma^2_t = \omega/(1-\beta) + \lambda(L)\varepsilon^2_t$, where $\lambda_k \approx k^{d-1}$, which is essentially the long memory property, or "Hurst effect" of hyperbolic decay. The attraction of the FIGARCH process is that for $0 < \delta < 1$, it is sufficiently flexible to allow for intermediate ranges of persistence. Analogous behavior in the conditional mean of exchange rates has been considered by Cheung (1993). The simpler FIGARCH(1,δ,0) process is of the form,

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \left[1 - \beta L - (1-L)^{\delta}\right]\varepsilon_t^2 .$$

The equations 1 through 3 are estimated by using non-linear optimization procedures to maximize the Gaussian log likelihood function,

$$\log(\mathcal{L}) = -(T/2)\ln(2\pi) - (1/2)\sum_{t=1,T}\left[\ln\left(\sigma_t^2\right) + \varepsilon_t^2\sigma_t^{-2}\right] \qquad (4)$$

with respect to the vector of parameters denoted by θ. Since most return series are not well described by the conditional normal density in (4), subsequent inference is consequently based on the Quasi Maximum Likelihood Estimation (QMLE) technique of Bollerslev and Wooldridge (1992), where

$$T^{1/2}\left(\hat{\theta}_T - \theta_0\right) \to N\left\{0, A(\theta_0)^{-1} B(\theta_0) A(\theta_0)^{-1}\right\},$$

and $A(.)$ and $B(.)$ represent the Hessian and outer product gradient respectively; $\hat{\theta}_T$ represents the estimates based on T observations, and θ_0 denotes the true parameter values.

Results of the estimated models for returns every day through seven days of temporal aggregation are presented in table 1. Hence the returns are computed every k days, where k = 1, 2,....7. The estimate of the long memory parameter, δ, for daily data is .38. This estimate is very close to a semi parametric estimate of the long memory parameter obtained for the absolute values of daily DM-$ returns by Andersen and Bollerslev (1997b). Various tests for specification of the daily model were performed.[1] In particular, a robust Wald test of a stationary GARCH(1,1) model under the null hypothesis versus a FIGARCH(1,δ,1) model under the alternative hypothesis has a numerical value of 25.37, which shows a clear rejection of the null when compared with the critical values of a chi squared distribution with one degree of freedom. Hence there is strong support for the hyperbolic decay and persistence as opposed to the conventional exponential decay associated with the stable GARCH(1,1) model. A sequence of diagnostic portmanteau tests on the standardized residuals and squared standardized residuals failed to detect any need to further complicate the model.[2]

Table 1 also shows that the estimates of δ are statistically significant at the .05 percentile for one through to seven days. In a recent study of ten years of high frequency DM-$ and Yen-$ returns, Andersen et al. (2000) have constructed model free measures of volatility for different temporal aggregations and conclude in favor of significant volatility

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1. The sample period for the daily returns model includes some periods of financial market crisis, such as the equity market meltdown of October 19, 1987 and the EMS crisis of September, 1992. Consistent with other studies, we regard these episodes as being part of the same generating process, rather than signalling a shift to a new regime. For this reason, we resist including dummy variables or any other mechanism of inducing a "better fit" to the sample period.

2. Tests of model diagnostics are performed by the application of the Box-Pierce portmanteau statistic on the standardized residuals. The standard portmanteau test statistic $Q_m = T\Sigma_{j=1,m}\, r_j^2$, where r_j is the j-th order sample autocorrelation from the residuals is known to have an asymptotic chi squared distribution with m–k degrees offreedom, where k is the number of parameters estimated in the conditional mean. Similar degrees of freedom adjustment are used for the portmanteau test statistic based on the squared standardized residuals when testing for omitted ARCH effects. This adjustment is in the spirit of the suggestions by Diebold (1988) and others.

TABLE 1. Estimated MA(1)-FIGARCH (p, δ, q) Models for Daily Returns

K	1	2	3	4	5	6	7
μ	.0028	–.0026	–.0006	.0116	.0104	.0671	–.0040
	(.0089)	(.0188)	(.0303)	(.0411)	(.0484)	(.0628)	(.0775)
θ	.0126	.0074	.0475	–.0005	.0791	.0993	.1256
	(.0150)	(.0206)	(.0264)	(.0270)	(.0313)	(.0367)	(.0403)
δ	.03826	.3475	.2665	.2907	.2395	.2370	.2323
	(.0760)	(.0916)	(.0753)	(.0839)	(.0740)	(.0750)	(.0841)
ω	.0207	.0865	.1944	.2309	.3962	.4831	.5865
	(.0088)	(.0311)	(.0811)	(.1010)	(.1826)	(.2302)	(.2789)
β	.5803	.2672	.2106	.2352	.1593	.1544	.1341
	(.0833)	(.0981)	(.0792)	(.0863)	(.0794)	(.0776)	(.0808)
φ	.2943	-	-	-	-	-	-
	(.0580)	-	-	-	-	-	-
ln(L)	–4,968.454	–3,388.357	–2,606.957	–2,162.524	–1,829.959	–1,598.865	–1,415.321
T/k.	4,989	2,494	1,663	1,247	997	831	712
Skewness	–.142	-.082	-0.021	0.030	-0.051	.027	-.039
Kurtosis	4.496	4.441	4.102	4.429	3.766	3.376	3.668
Q(20)	35.209	33.710	24.705	23.157	15.653	10.432	23.672
Q^2(20)	14.380	14.792	14.642	17.145	20.664	17.621	15.917

Note: The daily series is the DM/\$ spot exchange returns and is from March 14, 1979 through December 31, 1998; a total of T = 4,989 observations. The other series are observed every k days and contain T/k observations. QMLE asymptotic standard errors are in parentheses below corresponding parameter estimates. The quantity ln(L) is the value of the maximized log likelihood. The sample skewness and kurtosis refer to the standardized residuals. The Q(20) and Q^2(20) statistics are the Ljung-Box test statistics for 20 degrees of freedom to test for serial correlation in the standardized residuals and squared standardized residuals.

$$R_t = 100 \times \sum_{i=(t-1)k,tk} \left[\ln\left(S_i\right) - \ln\left(S_{i-1}\right)\right] = \mu + \varepsilon_t \theta \varepsilon_{t-1} ,$$

$$\varepsilon_t = \xi_t \sigma_t \text{ where } \xi_t \text{ is i.i.d (0,1) process,}$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \left[1 - \beta L - \left(1 - \varphi L\right)\left(1 - L\right)^{\delta}\right]\varepsilon_t^2 \text{, for } t = 1, ..., T/k, \text{ and } k = 1,....,7.$$

clustering, i.e. ARCH effects, for monthly data. Their finding contrasts with previous studies by Diebold (1988), Baillie and Bollerslev (1989) and Christoffersen and Diebold (1998), who tended to find that monthly exchange rate returns were close to being Gaussian and independently distributed. However, as noted by Andersen et al. (2000), their measure of integrated volatility should remain highly serially correlated even at a monthly level. The results reported in table 1 are consistent with the notion of self similar returns process with the same long memory volatility parameter, δ, up to seven days. Although the estimated δ parameter varies from .38 to .23 the range of values is well within the two robust standard error bands.[3]

III. Analysis of High Frequency Returns

This section is concerned with the set of 30 minute DM-$ spot exchange rate data provided by Olsen & Associates of Zurich, in which Reuter FXFX quotes are taken every 30 minutes for the complete calendar year of 1996. The sample period is 00:30 GMT, January 1, 1996 through 00:00 GMT, January 1, 1997. Each quotation consists of a bid and an ask price and is recorded in time to the nearest second. Following the procedures of Müller et al. (1990) and Dacorogna et al. (1993), the spot exchange rate for each 30 minute interval is determined as the linearly interpolated average between the preceding and the following quotes. Hence the 30 minute return series is defined as the difference between the midpoint of the logarithmic bid and ask rates. For example, if at time 0:30:00, the preceding bid-ask price pair is 1.4334-1.4341, and the following quote is 1.4330-1.4335, then the interpolated exchange rate ($S_{t,n}$) at 0:30:00 would be

$$S_{t,n} = \exp\{(1/2)\times[\ln(1.4334)+\ln(1.4341)] + (1/2)\times[\ln(1.4330)+\ln(1.4335)]\}. \tag{6}$$

Then the n-th 30 minute spot return for day t is, $R_{t,n} = \ln(S_{t,n}) - \ln(S_{t,n-1})$. It has become fairly standard in this literature to remove atypical data associated with slower trading patterns during weekends. Hence returns from Friday 21:00 GMT through Sunday 20:30 GMT are excluded. However, returns for holidays occurring during the sample are retained in order to preserve the number of returns associated with one week. In particular, the eventual sample used in subsequent analysis contains 262 trading days, each with 48 intervals of 30 minute duration; which realizes

3. Drost and Nijman (1993) have provided a theoretical treatment of the effect of temporal aggregation of an underlying high frequency GARCH(1,1) process. As yet no corresponding results exist for the FIGARCH process.

a total of 12,576 observations for the DM-$ returns for the 262 days.

Figure 2 plots the first 240 autocorrelation coefficients for the returns, squared returns and absolute returns of the unadjusted (raw) 30 minute DM-$ exchange rates for 1996. The usual $T^{-1/2}$ asymptotic standard errors for the sample autocorrelations are not strictly valid for a process with ARCH effects and are no more than useful guidelines. As usual there is a small, negative but very significant first order autocorrelation in returns, which may be due to the non-synchronous trading phenomenon,[4] while higher order autocorrelations are not significant at conventional levels. However, the autocorrelation functions of the squared and absolute returns exhibit a pronounced U shape pattern, associated with substantial intra day periodicity.[5] Similarly to the findings of Granger and Ding (1996), this pattern is particularly strong in absolute returns; and the general pattern is consistent with the studies of Wasserfallen (1989), Müller et al. (1990), Baillie and Bollerslev (1991), Dacorogna et al. (1993) and Andersen and Bollerslev (1998). The pattern is generally attributed to being due to the opening of the European, Asian and North American markets superimposed on each other. A further representation of this phenomenon is provided by figure 3, which shows the absolute 30 minute returns for each of the 48 intervals, averaged over all the days in the year. The highest average absolute returns occur between periods 26 and 34, which correspond to 1:00pm and 5:00pm GMT.

In order to remove the strong intra day periodicity, this study follows a similar approach as Andersen and Bollerslev (1998), and uses a two step estimation method, whereby the intra day periodicity is first removed by applying Gallant's (1981, 1982) FFF approach. In particular,

$$R_{t,n} = E\left(R_{t,n}\right) + \left(\sigma_t s_{t,n} z_{t,n} N^{-1/2}\right), \tag{7}$$

where $E(R_{t,n})$ is the unconditional mean of returns, σ_t is the conditional

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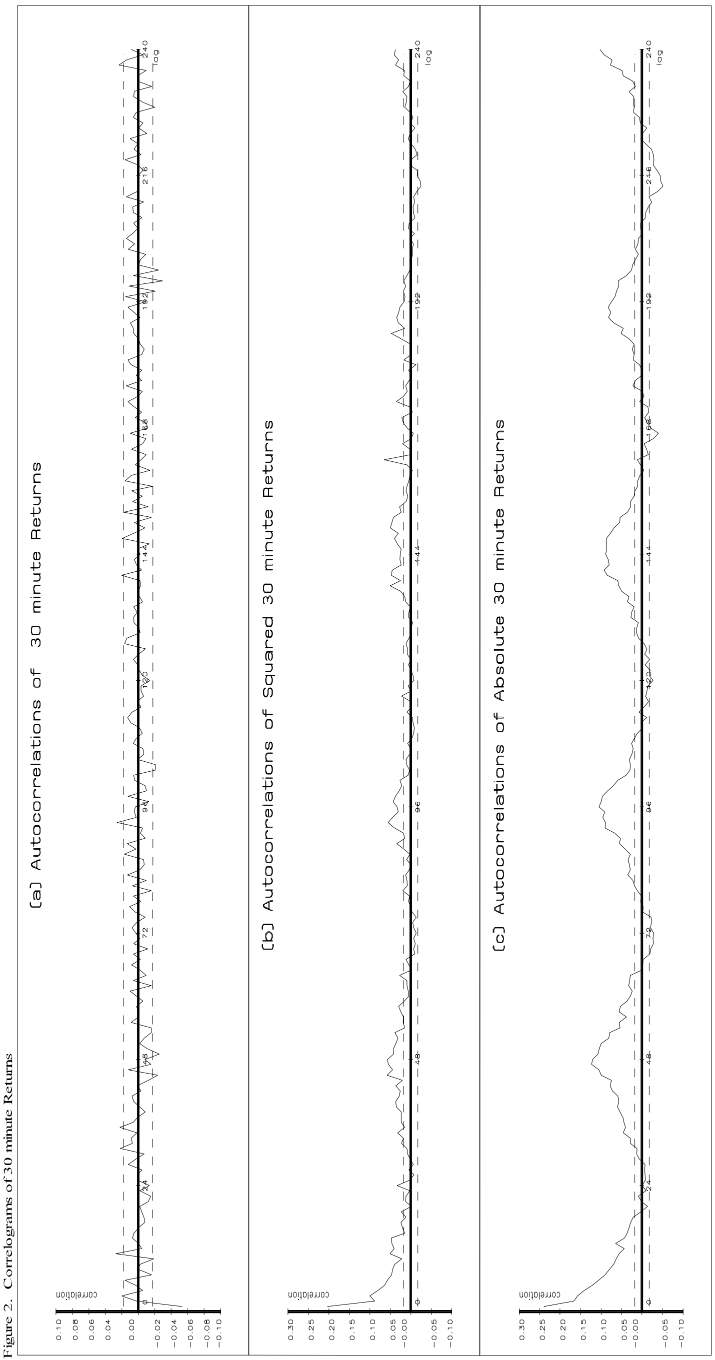
4. The small, but significant first order autocorrelation in high frequency data has also been noted by Andersen and Bollerslev (1997a), Goodhart and Figliuoli (1992), Goodhart and O'Hara (1997) and Zhou (1996).

5. Similar U-shaped patterns are found in the equity markets, see Harris (1986), Wood et al.(1985), Chang et al.(1995) and Andersen and Bollerslev (1997a).

Figure 2. Correlograms of 30 minute Returns



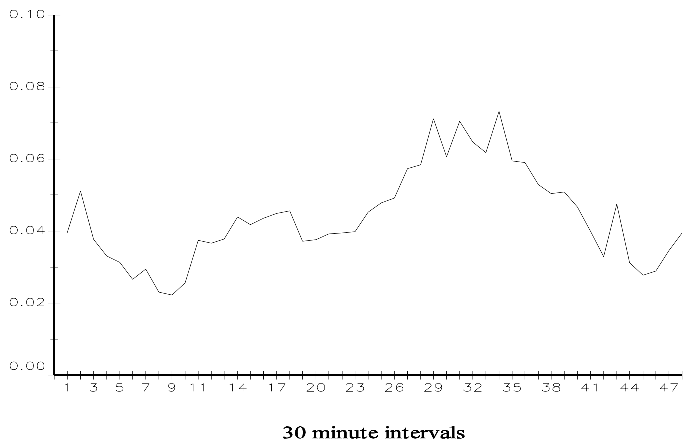


FIGURE 3.— Averages of absolute 30 minute returns. The vertical axis shows the absolute DM-$ returns averaged over the $T = 262$ days. The horizontal axis gives the period within the day.

variance of daily returns, $s_{t,n}$ is a deterministic function to represent intra day seasonality, $z_{t,n}$ is an i.i.d.(0,1) process, which is independent of the daily volatility process σ_t and N is the number of return intervals per day. From equation 7,

$$x_{t,n} = 2\ln\left|R_{t,n} - E\left(R_{t,n}\right)\right| - \ln\left(\sigma_t^2\right) + \ln\left(N\right) = \ln\left(s_{t,n}^2\right) + \ln\left(z_{t,n}^2\right),$$

where the observed variable $x_{t,n}$ is then regressed on a non linear function of the time interval n, and daily volatility σ_t; i.e.

$$x_{t,n} = f\left(\theta; t, n\right) + u_{t,n},$$

where

$$u_{t,n} = \ln\left(z_{t,n}^2\right) - E\left[\ln\left(z_{t,n}^2\right)\right],$$

is an i.i.d.(0,1) process and the functional form of $x_{t,n}$ is,

$$f\left(\theta; t, n\right) = \mu_0 + \mu_1 n / N_1 + \mu_2 n^2 / N_2 + \lambda_k I_k\left(t, n\right) + \sum_{k=1,3} \theta_k D_k\left(t, n-k\right) \tag{8}$$

$$+\sum_{p=1,k}\left[\delta_{c,p}\cos\left(p2\pi n/N\right)+\delta_{c,p}\sin\left(p2\pi n/N\right)\right],$$

where

$$N_1 = N^{-1}\sum_{i=1,N} i = \left(N+1\right)/2 \ ,$$

$$N_2 = N^{-1}\sum_{i=1,N} i^2 = \left(N+1\right)\left(2N+1\right)/6 \ ,$$

and $I_k(t,n)$ is an indicator variable which represents the occurrence of an event k on day t at interval n. These events include U.S. economic announcements of retail sales, trade balances, unemployment, and the PPI and CPI price indices. The indicator function is equal to unity when an announcement of the above occurs and is zero otherwise. After some experimentation it was also decided to include a lagged indicator variable, D_k, which is unity for the two hours immediately following the event and is zero otherwise. The dates and times for the U.S. economic announcements were obtained from the section "Week Ahead" of the weekly magazine, *Business Week*. On treating the variable $x_{t,n}$ as the dependent variable, the parameters in the equation 8 were estimated by OLS. The intra day seasonality for interval n, on day t is then estimated as

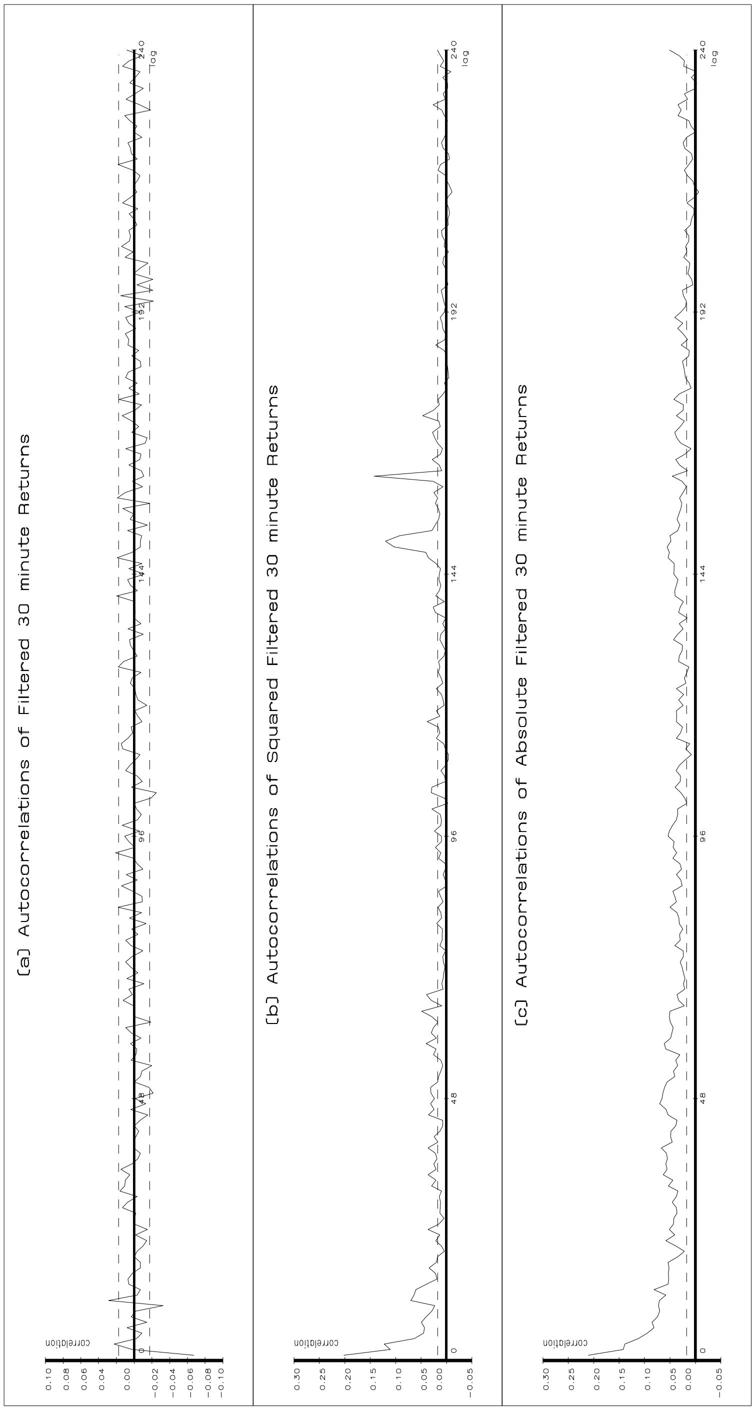
$$s_{t,n} = \frac{T.\left[\exp\left(f_{t,n}/2\right)\right]}{\sum_{t=1(T/N)}\sum_{n=1,N}\exp\left(f_{t,n}/2\right)} . \qquad (9)$$

The 30 minute returns are then filtered by the estimated intra day seasonality series, $s_{t,n}$, to generate the filtered returns, which are defined as

$$\tilde{R}_{t,n} = R_{t,n}/s_{t,n} . \qquad (10)$$

Figure 4 presents the autocorrelations of the raw, squared and absolute filtered 30 minute DM-$ returns. It is clear that the autocorrelations of the squared and absolute filtered returns have dramatically reduced intra

Figure 4. Correlograms of filtered 30 minute returns.



day periodicity, while also exhibiting extreme persistence associated with the feature of long memory.

A generalization of the daily model, which is also based on the continuously compounded returns for sampling frequency *k*, includes an MA(1)-FIGARCH(1,δ,1) formulation,

$$\tilde{R}_{t,n} = \mu + \varepsilon_{t,n} + \theta\varepsilon_{t,n-1}, \tag{11}$$

$$\varepsilon_{t,n} = z_{t,n}\sigma_{t,n}, \tag{12}$$

$$\sigma^2_{t,n} = \omega + \beta\sigma^2_{t,n-1} + \left[1 - \beta L - (1-\varphi L)(1-L)^{\delta}\right]\varepsilon^2_{t,n}, \tag{13}$$

where $z_{t,n}$ is an i.i.d.(0,1) process, and where the two time indices are t = 1,.. 262 days, and n = 1,.. K, intra day periods, so that $K = 48/k$, for k = 1, 2, 3, 4, 6, 8, 12 and 16. Results for estimating the above model for the six different frequencies over *k*, within the day are presented in table 2. The estimated long memory volatility parameter, δ, is estimated within the range of .14 to .24 for the 30 minute, one hour, 90 minutes, two hour, three hour, four hour, six hour and eight hour sampling frequencies. The estimate of δ is highly significant for all the returns series.

In many previous studies the GARCH(1,1) model has been used to represent the volatility process. The results in table 2 indicate that the FIGARCH model is generally the more appropriate specification. For the 30 minute return series, a robust Wald test of the stationary GARCH(1,1) null hypothesis versus a FIGARCH(1,δ,1) alternative hypothesis has a numerical value of 63.00, which provides an overwhelming rejection of the GARCH(1,1) formulation. Hence, as with the daily data, there the tests imply strong support for the long memory FIGARCH model. Tests for mis-specification do not reveal any obvious deficiencies with the model, and in particular the standardized residuals from the MA(1)-FIGARCH(1,δ,0) model for the filtered 30 minute returns appear to be uncorrelated. The estimates of the long memory parameter are relatively stable across the high frequency data from 30 minutes to 8 hours; and in this sense are comparable with the semi parametric estimates of long memory from absolute returns obtained by Andersen and Bollerslev (1998).

An interesting implication of the robustness of the FIGARCH model

TABLE 2. Estimated MA(1)-FIGARCH(p,δ,q) Model for Filtered 30-minute Returns

	30min.	1 hour	1.5 hours	2 hours	3 hours	4 hours	6 hours	8 hours
k	1	2	3	4	6	8	12	16
μ	.0006	.0014	.0022	.00037	.0048	.0065	.0104	.0106
	(.0005)	(.0009)	(.0014)	(.0019)	(.0030)	(.0040)	(.0059)	(.0079)
θ	–.0936	–.0494	.0016	.0052	–.0081	.0341	.0143	–.0314
	(.0113)	(.00151)	(.0196)	(.0235)	(.0315)	(.0291)	(.0415)	(.0367)
δ	.2049	.1914	.2031	.1504	.1824	.1416	.2453	.2396
	(.0257)	(.0413)	(.0473)	(.0437)	(.0811)	(.0522)	(.1395)	(.1910)
ω	.0009	.00019	.0025	.00046	.0063	.0098	.0093	.0046
	(.0001)	(.0004)	(.0007)	(.0014)	(.0022)	(.0034)	(.0043)	(.0050)
β	.0354	.0294	.0755	.0058	.1028	.0428	.0787	.7231
	(.0314)	(.0526)	(.0521)	(.0715)	(.1084)	(.0714)	(.1419)	(.1923)
φ	-	-	-	-	-	-	-	.6017
	-	-	-	-	-	-	-	(.2422)
ln(L)	17,622.853	6,788.983	3,714.431	2,353.932	1,081.756	618.046	164.361	10.751
T	12,576	6,288	4,192	3,144	2,096	1,572	1,048	786
Skewness	–.183	.047	–.165	.152	–.922	.028	–.474	–.352
Kurtosis	9.236	8.321	7.731	8.037	18.774	6.761	7.732	6.533
Q(50)	67.984	72.163	66.752	70.820	59.907	69.127	38.847	49.513
Q^2(50)	27.134	61.351	24.737	49.173	24.714	69.154	35.283	31.193

Note: The 30-minute spot exchange returns are from 00:30 GMT, January 1, 1996 through 00:00 GMT, January 1, 1997 for a total of 262 days. The total number of observations is T = 262.k, for k=1, 2, 3, 4, 6, 8,12, and 16. The intraday periodicity ($\hat{s}_{t,i}$) is estimated by the FFF(Flexible Fourier Form) method. QMLE asymptotic standard errors are in parentheses below corresponding parameter estimates. The quantity ln(L) is the value of the maximized log likelihood. The sample skewness and kurtosis refer to the standardized residuals. The Q(50) and Q^2(50) statistics are the Ljung-Box test statistics for 50 degrees of freedom to test for serial correlation in the standardized residuals and squared standardized residuals.

$$\tilde{R}_t = 100 \times \sum_{i=(n-1)k+1,nk} \left[R_{t,n} / \hat{S}_{t,i-1} \right] = \mu + \varepsilon_{t,n} \theta \varepsilon_{t,n-1} ,$$

$$\varepsilon_t = \xi_t \sigma_t \text{ where } z_{t,n} \text{ is i.i.d (0,1) process,}$$

$$\sigma_{t,n}^2 = \omega + \beta \sigma_{t,n-1}^2 + \left[1 - \beta L - (1 - \varphi L)(1 - L)^{\delta} \right] \varepsilon_{t,n}^2 ,$$

for t = 1,...,262 days, and n = 1,...., K, where K=48/k and k = 1,...,8.

and importance of the long memory volatility parameter on relatively short spans of high frequency data, strongly suggests that the long memory property is an intrinsic feature of the system rather than being due to exogenous shocks which lead to regime shifts.

IV. Nonlinear Dependencies in High Frequency Returns

The above model has used the FFF adjustment to remove intra day periodicity and has used the MA(1)-FIGARCH(1,δ,0) process to represent the high frequency returns. However, it is still possible that higher order non linearities are present in the high frequency returns and are not captured by the model. This possibility is enhanced by the use of the FFF procedure being applied to the raw returns. Hence it is of interest to apply further tests for omitted non linear effects. One way of searching for temporal dependence in time series data is the estimation of the correlation dimension, which is based on the notion of correlation integral. For a given time series $\{x(t)$: t = 1,....$T\}$ of D dimensional vectors, the so-called correlation integral $C(\varepsilon)$ is defined as

$$C(\varepsilon) = \lim_{t \to \infty} \left[2/T(T-1) \right] \sum I_{\varepsilon}(x_i, x_j),$$

where $I_{\varepsilon}(x,y)$ is an indicator function which is equal to one if $||x–y|| < \varepsilon$, and is zero otherwise, and where $||.||$ denotes the sup-norm. Intuitively, the correlation integral $C(\varepsilon)$ measures the fractions of the pairs of points $\{x(t)\}$ that are within an ε distance from each other. Grassberger and Procaccia (1983) define the correlation dimension of the time series $\{x(t)\}$ of an embedding dimension M as,

$$D^M = \lim_{\varepsilon \to 0} \left[\ln \{C(\varepsilon)\} / \ln(\varepsilon) \right].$$

The correlation dimension technique produces estimates and uses graphical analysis, which typically requires very large data sets, that are more common in physics, as opposed to economics and finance. The technique has been implemented on economic data by Ramsey et al. (1990) and others. As noted by Hsieh (1989) and Ramsey et al. (1990), the correlation dimension estimated from small sample sizes can be very misleading, and has a downwards bias, thus increasing the probability of

TABLE 3. Tests for Non Linearity in Model's Residuals

A. Correlation Dimensions

M	$C(M)$
1	0.69
2	2.07
3	2.99
4	3.75
5	4.30
6	4.82
7	5.19
8	5.48
9	5.67
10	5.80

B. BDS Statistics

ε	.25	.5	.75	1	1.25
M = 2	–.46	–.12	–.67	–.07	.12
M = 3	–.51	–.04	–.07	.03	.30
M = 4	–.12	–.13	–.25	–.10	.19
M = 5	1.39	–.13	–.36	–.25	–.03
M = 6	3.79	–.07	–.31	–.30	–.15
M = 7	7.72	–.27	–.35	–.31	–.17
M = 8	23.30	–.27	–.44	–.35	–.20
M = 9	63.81	–.08	–.44	–.39	–.24
M = 10	198.27	.54	–.47	–.43	–.34

Note: The correlation dimensions are for the residuals from the *MA*(1) - FIGARCH(1,δ,0) model for the filtered 30 minute DM-US-$ returns. The M symbol denotes the embedding dimension and $C(M)$ represents the corresponding correlation dimension. The above BDS statistics are for an embedding dimension M with distance of ε; and are for the same residuals as in panel A.

erroneously concluding in favor of finding low-dimensional chaos. Panel A of table 3 reports the correlation dimension estimates for the residuals from the model given by equations 11 through 13 for the 30 minute returns. Following Liu et al. (1992) and Cecen and Erkal (1996), concerning the interpretation of these estimates, it is clear that there is little convergence in dimension estimates, and they do not appear to indicate any strong low dimensional deterministic dependence in the residuals. A further test for nonlinear dependence is the BDS test, of Brock et al. (1996). This test attempts to distinguish between an i.i.d. series and a series with deterministic or stochastic dependence. Given a time series x_t, for $t = 1,....T$ which is an i.i.d. sequence, it can be shown that

$$C_n(\varepsilon) = C_1(\varepsilon)^n,$$

where $C_n(\varepsilon)$ is the correlation integral. By estimating $C_1(\varepsilon)$ and $C_n(\varepsilon)$ by the sample values $C_{1,T}(\varepsilon)$ and $C_{n,T}(\varepsilon)$, the BDS test statistic can be written as,

$$B_{n,T}(\varepsilon) = \frac{T^{1/2}\left[C_{n,T}(\varepsilon) - C_{1,T}(\varepsilon)^n\right]}{\sigma_{n,T}(\varepsilon)}, \tag{14}$$

where $\sigma_{n,T}(\varepsilon)$ is an estimate of the asymptotic standard error of the numerator in equation 14. Then under the i.i.d. null hypothesis, Brock et al. (1996) prove that $B_{n,T} \sim N(0,1)$. The embedding dimension, M, was chosen to be in the range of 2 through 10, while ε was fixed in the range of .25s through 1.25s, where s is the standard deviation of the data. Panel B of table 3 reports the BDS test results for the residuals from the estimated model in equations 11 through 13 from the 30 minute filtered returns. Except for the value of ε = .25 and $M \geq 6$, the test statistics consistently fail to reject the i.i.d. null for the residuals. Overall there is no evidence from these tests for non linearity to indicate model mis-specification. Hence there is no reason to doubt that the FFF filtering procedure and the estimated MA(1)-FIGARCH(1,δ,0) model has adequately captured the dynamics, both linear and nonlinear, of the high frequency exchange rate returns.

V. Conclusion

This paper has considered one year of high frequency DM-$ returns, and also twenty years of daily and lower frequency data. The 30 minute returns series were filtered by the Flexible Fourier Form (FFF) method to remove intra day periodicity. The long memory volatility processes, FIGARCH, is found to provide a good representation of both the high frequency and the daily DM-$ returns data. Two tests for non linearity are presented to further test the appropriateness of the FFF filtering procedure and also the imposition of the MA-FIGARCH model. The residuals from the model are found to be free of any significant non linear effects, and the FIGARCH model appears to successfully account for the dynamics of the return series. Interestingly, the estimates of the long memory volatility parameter in the FIGARCH models are very close across time aggregations, suggesting that long memory volatility is an intrinsic feature of the system.

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